Local search methods for a single machine scheduling problem

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Abstract

The problem of scheduling jobs on a single machine to minimize the weighted sum of squares completion time is considered .Al-Salihi used a branch and bound method to minimize the weighted sum of squares completion time. In this research we discuss and apply some known local search methods , namely the adjacent pairwise interchange method (APIM) and descent method (DM) . The performance of local search methods can be tested on large class of test problems.

Keywords :Heuristics , local search, optimization

الملخص: في هذا البحث نتناول مسألة جدولة نتاجات على ماكنة واحدة لتصغير دالة الهدف وهي المجموع الوزني لمربعات وقت الاتمام (The weighted sum of squares completiom time). درست هذه المسألة من قبل Al-Salihi حيث استعمل طريقة التقيد والتفرع (Branch and bound method). في هذه البحث نناقش ونطبق بعض طرائق تقريبات البحث المحلي (Local search methods) المعروفة وهي(APIM) (Adjacent pairwise interchange method) و (DM) (Descent method) . يمكن تطبيق طرائق البحث المحلي على عدد كبير من مسائل الاختبار.

١-Introduction

In this research we use local search methods for the problem of scheduling n jobs on a single machine . Al-Salihi [١] used a branch and bound method to find an optimal solution. Local search methods are an iterative improvement techniques and more recently developed meta-heuristics . It is a generally applicable approach that can be used to find approximate solutions to hard optimization problems[٥].It works as starting with some feasible initial solution , a neighbour (i.e. a feasible solution) in some predefined neighbourhood is generated and then the objective function value of this generated neighbour is compared with that of the starting solution . By means of some acceptance criterion , it is decided which of both feasible solutions is selected to be the starting solution for the next neighbour generation [٤].

٢-Problem Definition

The machine scheduling problem which we consider can be described as follows : There are n jobs (numbered ١,…,n). All jobs are available for processing at time zero and are to be schedule on a single machine [٢], each job i has a given processing time Pi and an associated positive weight Wi . The objective is to find a processing order of the jobs that minimizes the weighted sum of squares completion time , the objective function defined by $\sum_{i=١}^{n} w_i C_i^{٢}$ where C_i is the completion time of job i .

٣- Representation and Neighbourhoods:

We now discuss how solutions are represented and the choice of neighbourhood. The definition of moves in a neighbourhood is related to the representation of a solution .For sequencing problems , the natural representation of a solution is a permutation of the integers ١،٢,…..,n
with n the number of jobs . On this representation two basic neighbourhoods can be defined [٣]. Each is illustrated by considering a typical neighbour of the sequence (١،٢،٣،٤،٥،٦،٧) in a problem where there are seven jobs labeled ١،٢,….,٧.

(١) Swap neighbourhood

a) Swap two jobs which is not adjacent . Thus , (١،٢،٦،٤،٥،٣،٧)is a neighbour of the solution.

b) Swap two adjacent jobs .Thus ,(١،٣،٢,٤،٥،٦،٧) is a neighbour of the solution.

(٢) Insert neighbourhood

a) Remove a job from one position in the sequence and insert it at another position (either before or after the original position).Thus, (١،٤،٢،٣،٥،٦،٧) and (١،٢،٣،٥،٦،٤،٧) are both neighbours.

b) Move a subsequence of jobs from one position in the sequence and insert it at another position. Thus ,(١،٤،٥،٢،٣،٦،٧) is a neighbor.

٤- An Adjacent Pairwise Interchange Method (APIM)

The adjacent pairwise interchange API is obtained as follows:

Step (١):An initial feasible solution is obtained by applying the shortest weighted processing time (SWPT) rule, in which the jobs are sequenced in non- decreasing order of processing time to weight ratios.
Hence ,we get the sequence σ = (σ (١), σ(٢) ,…., σ(n)), for this sequence we calculate the weighted sum of squares completion time and suppose its value (UB)is the current solution i.e. UB = $\Sigma^{n}_{i=١}$ w_i $C_i^{٢}$.

Step (٢): In order to improve the sequence σ , two adjacent jobs σ(i) and σ(j) are temporarily sequenced in position σ(j) and σ(i) (١≤ i≤j ≤ n) respectively. Hence, a new schedule σ٣ is obtained , for this new schedule we compute $UB_{٣} = \Sigma^{n}_{i=١}$ w_i $C_i^{٢}$.
Step (٣): If the improvement is made (i.e. , $UB_{٣}$ < UB) , then the two jobs σ(i) and σ(j) are left in their new positions . On the other hand , no improvement can be made , the two jobs σ(i) and σ(j) are replace in their original positions (i.e. position i and position j).
The procedure is then repeated from the beginning (i.e. i=١ and j= ٢) and other possibilities are considered in a similar way by repeating step (٢).
Step (٤) : The method terminates when all possibilities [(i= ١ ,…….,n-١) and (j=i+١ ,…..,n)] are considered without making any improvement.
To illustrate the above method , we present this example in which the optimal solution is ٣٨٦٥٠.
Example :
The problem of ١٠ jobs with the following data:-

i	١	٢	٣	٤	٥	٦	٧	٨	٩	١٠
Pi	٣	٥	٥	٥	٥	١	٩	١٠	٤	١٠
wi	٢	٣	٧	٢	٢	٣	٩	٩	١٠	٤

Step (١): Using the shortest weighted processing time (SWPT) rule , in which the jobs are sequenced in non- decreasing order of Pi / wi to obtain a sequence :

i	٦	٩	٣	٧	٨	١	٢	٤	٥	١٠
Pi	١	٤	٥	٩	١٠	٣	٥	٥	٥	١٠
wi	٣	١٠	٧	٩	٩	٢	٣	٢	٢	٤
Ci	١	٥	١٠	١٩	٢٩	٣٢	٣٧	٤٢	٤٧	٥٧

$Ci^{٢}$	١	٢٥	١٠٠	٣٦١	٨٤١	١٠٢٤	١٣٦٩	١٧٦٤	٢٢٠٩	٣٢٤٩
$wiCi^{٢}$	٣	٢٥٠	٧٠٠	٣٢٤٩	٧٥٦٩	٢٠٤٨	٤١٠٧	٣٥٢٨	٤٤١٨	١٢٩٩٦

And the initial current solution is UB = ٣٨٨٦٨ .

Step (٢): Two adjacent jobs ٦ and ٩ are temporarily sequenced in position ٩ and ٦ to obtain a new sequence (٩،٦،٣،٧،٨،١،٢،٤،٥،١٠), for this new sequence we compute $UB_{٣}$ = ٣٨٨٥٠.

Step (٣): For the last sequence the improvement is made ($UB_{٣}$ <UB) , then the two jobs ٩ and ٦ are left in their new positions .The procedure is repeated from the beginning and other possibilities are considered in a similar way by repeating step ٢ , then we get the following :

IT	Sequence	$UB_{٣}$ value
٠	(٦،٩،٣،٧،٨،١،٢،٤،٥،١٠)	٣٨٨٦٨
١	(٩،٦،٣،٧،٨،١،٢،٤،٥،١٠)	٣٨٨٥٠
٢	(٩،٣،٦،٧،٨،١،٢،٤،٥،١٠)	٣٨٩٤٢
٣	(٩،٦،٧،٣،٨،١،٢،٤،٥،١٠)	٣٩١٩٢
٤	(٩،٦،٣،٨،٧،١،٢،٤،٥،١٠)	٣٩٢٠١
٥	(٩،٦،٣,٧،١،٨،٢،٤،٥،١٠)	٣٩٤١٧
٦	(٩،٦،٣،٧،٨،٢،١،٤،٥،١٠)	٣٨٩٠١
٧	(٩،٦،٣،٧،٨،١،٤،٢،٥،١٠)	٣٩٢٤٥
٨	(٩،٦،٣،٧،٨،١،٢،٥،٤،١٠)	٣٨٨٥٠
٩	(٩،٦،٣،٧،٨،١،٢،٤،١٠،٥)	٣٨٧٥٠

IT: Number of iterations.

Step (٤): The method is terminates and we get the best sequence (٩،٦،٣،٧،٨,١،٢،٤،١٠،٥) and its objective function value is ٣٨٧٥٠.

٥- Descent Method (DM)

The simplest neighbourhood methods is a descent method (DM) that is obtained as follows :

Step (١): An initial feasible solution is obtained by applying the shortest weighted processing time (SWPT) rule, we get the sequence σ=(σ(١), σ(٢),……, σ(n)),and suppose its value (UB) is the current solution i.e. UB = $\Sigma^{n}_{i=١}$ w_i $C_i^{٢}$.
Step(٢): In order to improve the sequence σ, we search of the new neighbourhood of this sequence for a better one .two jobs σ(i) and σ(j) in this sequence are selected for consideration in a swap neighbourhood , random selection of σ(i) and σ(j) is applied and we interchange their positions into σ(j) and σ(i) . Hence , a new schedule σ١ is obtained , for this new schedule we compute UB١ = $\Sigma^{n}_{i=١}$ wiCi^{٢} .
Step(٣): The acceptance test chooses to be replace UB as the current solution if UB١< UB then set UB = UB١ and σ = σ١, otherwise UB is retained as the current solution.
Step (٤): Repeat step (٢) , and other possibilities are considered in a similar way. The method terminates if no neighbour provides an improved objective function value , in which case the current solution UB is a local optimum.

٦- Conclusions

Considering APIM and DM as approximate methods for obtaining good ,though not necessarily optimal solutions to the problem of scheduling a single machine to minimize the weighted sum of squares completion time. The local search methods have several attractive features . First of all , they are very easy to implement .Once a neighbourhood structure has been devised .Secondly they can be generally applied to a wide range of problems.

An interesting future research topic would involve experimentation with the following approximation algorithms **:** Tabu search and genetic algorithm**.**

References

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